Consolidation: Oblique asymptotes	
1.	Find the equations of the asymptotes of: $y=\dfrac{2x^2}{1-x}$
2.	Sketch a fully-labelled graph of: $y=\dfrac{2x^2-1}{2x+3}$
3.	Find the equations of the asymptotes and hence sketch: $y=\dfrac{x^3-x}{x^2-4}$ *(Bonus points if you can find the stationary points! You'll need to use calculus - can you see why the FM method using intersections with $y=k$ won't work here? If you haven't covered the quotient rule in your normal A Level Maths yet then don't worry, you can skip this extra part.)*

Reasoning

A curve with equation $y=\dfrac{ax^2+bx-1}{x+1}$ has an asymptote $y=4x-2$

a Find the values of a and b

b Write down the equation of the other asymptote.

c Without using calculus, find the coordinates of the turning points.

d Sketch the curve.

<u>**Consolidation: Complex powers and roots**</u>

Knowledge Check	
1.	Write in exponential form: $4\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$ $3\left(\cos\left(\frac{5\pi}{6}\right)-i\sin\left(\frac{5\pi}{6}\right)\right)$ $-4-8i$ Write in the form $a+bi$: $7e^{-\frac{\pi}{3}i}$ Given that $z=5e^{\frac{2\pi}{7}i}$ and $w=\frac{1}{5}e^{-\frac{\pi}{7}i}$, calculate the value of **a** $\lvert zw\rvert$ **b** $\left\lvert\frac{z}{w}\right\rvert$ **c** $\arg(zw)$ **d** $\arg\left(\frac{z}{w}\right)$
2.	Given that $z=4\left(\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right)$, express in exact Cartesian form **a** z^2 **b** z^3 **c** $\frac{1}{z}$ **d** $16z^{-4}$ Given that $z=-\sqrt{3}+i$, use de Moivre's theorem to write the following in Cartesian form. **a** z^4 **b** z^{-3} **c** z^{-2} **d** $\frac{8}{z^6}$
3.	Solve each of these equations, giving your solutions in exponential form **a** $z^3=4\sqrt{2}+4\sqrt{2}i$ **b** $z^3=-4\sqrt{2}+4\sqrt{2}i$ **c** $z^3=-4\sqrt{2}-4\sqrt{2}i$ **d** $z^3=4\sqrt{2}-4\sqrt{2}i$

Reasoning

A complex number z has modulus 1 and argument θ

a Show that $z^n+\frac{1}{z^n}=2\cos(n\theta)$ **b** Show that $z^n-\frac{1}{z^n}=2i\sin(n\theta)$

Given that ω is a complex cube root of unity

a Show that $1+\omega+\omega^2=0$

b Evaluate the following expressions.

i $(1+\omega)^2-\omega$ **ii** $(1+\omega)(1+\omega^2)$ **iii** $\omega(\omega+1)$ **iv** $\frac{2\omega+1}{\omega-1}+\omega$

Challenge

The points A, B and C represent the solutions to the equation $z^3=-27i$

a Find the solutions to the equation in the form $a+bi$

b Calculate the exact

i Area, **ii** Perimeter of triangle ABC

Consolidation: Further vectors

Knowledge Check

1.

Use the distributive and anticommutative properties of the vector product to show that

a $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{c}=(\mathbf{b}+\mathbf{a})\times(\mathbf{c}-\mathbf{a})$

b $\mathbf{b}\times(2\mathbf{a}+\mathbf{c})-\mathbf{c}\times(\mathbf{b}-\mathbf{c})=2\mathbf{b}\times(\mathbf{a}+\mathbf{c})$

Find a unit vector which is perpendicular to both $\begin{pmatrix}-5\\0\\0\end{pmatrix}$ and $\begin{pmatrix}-2\\1\\-3\end{pmatrix}$

Find the values of a, b and c given that

$$\begin{pmatrix}3\\-4\\a\end{pmatrix}\times\begin{pmatrix}0\\2\\b\end{pmatrix}=\begin{pmatrix}10\\9\\c\end{pmatrix}$$

Calculate the exact area of the triangles with vertices at $(1, 1, -2)$, $(0, 1, -1)$ and $(-2, 0, 1)$

2.

A plane contains the points $(5, 1, 1)$, $(-2, 0, 0)$ and $(6, 2, -5)$

Find the equation of the plane in

a Vector form, **b** Scalar product form,

c Cartesian form.

Calculate the acute angle between the line $\mathbf{r}=2\mathbf{i}-8\mathbf{j}+\mathbf{k}+t(\mathbf{j}+\mathbf{k})$ and the plane $\mathbf{r}\cdot(6\mathbf{i}-\mathbf{j}+5\mathbf{k})=9$

3.

Find the shortest distance between each point and plane.

a $(2, 5, 1)$ and $2x-4y+4z+5=0$

Find the point of intersection of the line with equation $\frac{x-3}{4}=\frac{y-2}{5}=\frac{z-1}{-3}$ and the plane with equation $\mathbf{r}=13\mathbf{i}+7\mathbf{j}-6\mathbf{k}+s(\mathbf{i}+2\mathbf{j}-5\mathbf{k})+t(-\mathbf{i}+\mathbf{j}+2\mathbf{k})$

Reasoning

The line l passes through the points $A(1, 4, -2)$ and $B(0, 2, -7)$

The plane Π has equation $5x-5y+z=3$

a Find the shortest distance from the plane to the point

i A **ii** B

b Does the line intersect the plane? Explain your answer.

Do these equations represent the same line? Explain how you know.

A: $\frac{x-3}{-2}=\frac{y+1}{4}=\frac{z-5}{-6}$

B: $\left(\mathbf{r}-\begin{pmatrix}4\\-3\\8\end{pmatrix}\right)\times\begin{pmatrix}-3\\6\\-9\end{pmatrix}=\mathbf{0}$

Challenge

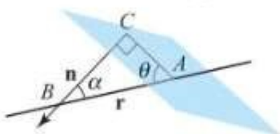
Three lines have equations as follows:

L_1: $\mathbf{r}=6\mathbf{i}-3\mathbf{j}+\lambda(\mathbf{i}+\mathbf{k})$,

L_2: $\mathbf{r}=s(3\mathbf{i}-\mathbf{j}+\mathbf{k})$ and

L_3: $\mathbf{r}=t(\mathbf{j}+2\mathbf{k})$

The lines L_1 and L_2 intersect at the point A, L_1 and L_3 intersect at the point B and L_2 and L_3 intersect at the point C, as shown.



Calculate the exact area of triangle ABC

Consolidation: Further matrices

Knowledge Check	
1.	$\begin{pmatrix} 2a & 3 & a \\ -b & 0 & b \\ 3 & 5 & 1 \end{pmatrix}$ $\begin{pmatrix} -4a & 2 & 1 \\ a & 0 & 2 \\ 3a & -1 & 1 \end{pmatrix}$ **a** Use one or more column operations to write $\begin{vmatrix} 2 & 1 & -3 \\ 4 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ as a determinant with at least two zero elements. **b** Hence find $\det\begin{pmatrix} 2 & 1 & -3 \\ 4 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ Use row or column operations to show that **a** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)(ab+ac+bc-a^2-b^2-c^2)$ **c** $\begin{vmatrix} a+b & a+c & b+c \\ c & b & a \\ c^2 & b^2 & a^2 \end{vmatrix} = (b-a)(a-c)(c-b)(a+b+c)$ **b** $\begin{vmatrix} a^2 & b^2 & c^2 \\ bc & ca & ab \\ 1 & 1 & 1 \end{vmatrix} = (b-a)(c-a)(c-b)(a+b+c)$ **d** $\begin{vmatrix} a & -b & c \\ c-b & a+c & a-b \\ -bc & ac & -ab \end{vmatrix} = (a+b)(c-a)(b+c)(a-b+c)$ Find the determinant of: Find the inverse of:
2.	$x+y-2z=3$, $2x-3y+5z=4$, $5x+2y+z=-3$ $3x-2y+z=7$, $6x-4y+2z=5$, $x+3y+2z=3$ $x+2z=1$, $-2x+y+4z=0$, $9x-2y+2z=5$ Describe fully the geometric arrangements of the following systems of planes:
3.	$\begin{pmatrix} 1 & 0 & -5 \\ 4 & 5 & 1 \\ 0 & 0 & -3 \end{pmatrix}$ Find the eigenvalues and eigenvectors of: Given $\mathbf{B} = \begin{pmatrix} 2 & 6 \\ 1 & -3 \end{pmatrix}$, write $\mathbf{B}$ in the form $\mathbf{B} = \mathbf{PDP}^{-1}$

Reasoning

Given that the determinant of $\begin{pmatrix} a & 2 & -3 \\ 1 & a & 4 \\ 3 & 2 & 5 \end{pmatrix}$ is 128, calculate each of these determinants. Justify your answer in each case.

a $\begin{vmatrix} 1 & a & 4 \\ a & 2 & -3 \\ 3 & 2 & 5 \end{vmatrix}$

e $\begin{vmatrix} a & 6 & -3 \\ 1 & 3a & 4 \\ 3 & 6 & 5 \end{vmatrix}$

c $\begin{vmatrix} a+1 & 2+a & 1 \\ 1 & a & 4 \\ 3 & 2 & 5 \end{vmatrix}$

g $\begin{vmatrix} -3 & 2a & 2 \\ 4 & 2 & a \\ 5 & 6 & 2 \end{vmatrix}$

Challenge

Given that $\mathbf{T} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

a Work out $\mathbf{T}^5$

b Show that $\mathbf{T}^n = 5^{n-1}\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

Review: MEI AS Further Maths 2021 Papers

These questions are from the MEI exam board. They are sometimes a little different in style, but cover the same content.

Core pure

1 Using standard summation formulae, find $\sum_{r=1}^{n}(r^2 - 3r)$, giving your answer in fully factorised form. **[3]**

2 The equation $3x^2 - 4x + 2 = 0$ has roots α and β.

Find an equation with integer coefficients whose roots are $3 - 2\alpha$ and $3 - 2\beta$. **[3]**

3 Three planes have the following equations.

$$\begin{aligned} 2x - 3y + z &= -3, \\ x - 4y + 2z &= 1, \\ -3x - 2y + 3z &= 14. \end{aligned}$$

(a) **(i)** Write the system of equations in matrix form. **[1]**

(ii) Hence find the point of intersection of the planes. **[2]**

(b) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $2x - 3y + z = -3$ and $x - 4y + 2z = 1$. **[4]**

4 Anika thinks that, for two square matrices $\mathbf{A}$ and $\mathbf{B}$, the inverse of $\mathbf{AB}$ is $\mathbf{A}^{-1}\mathbf{B}^{-1}$. Her attempted proof of this is as follows.

$$\begin{aligned} (\mathbf{AB})(\mathbf{A}^{-1}\mathbf{B}^{-1}) &= \mathbf{A}(\mathbf{BA}^{-1})\mathbf{B}^{-1} \\ &= \mathbf{A}(\mathbf{A}^{-1}\mathbf{B})\mathbf{B}^{-1} \\ &= (\mathbf{AA}^{-1})(\mathbf{BB}^{-1}) \\ &= \mathbf{I} \times \mathbf{I} \\ &= \mathbf{I} \end{aligned}$$

Hence $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$

(a) Explain the error in Anika's working. **[2]**

(b) State the correct inverse of the matrix $\mathbf{AB}$ and amend Anika's working to prove this. **[3]**

5 Prove by induction that $\sum_{r=1}^{n} r \times 2^{r-1} = 1 + (n-1)2^n$ for all positive integers n. **[5]**

6 A transformation T of the plane has associated matrix $\mathbf{M} = \begin{pmatrix} 1 & \lambda+1 \\ \lambda-1 & -1 \end{pmatrix}$, where λ is a non-zero constant.

(a) **(i)** Show that T reverses orientation. **[3]**

(ii) State, in terms of λ, the area scale factor of T. **[1]**

(b) **(i)** Show that $\mathbf{M}^2 - \lambda^2\mathbf{I} = \mathbf{0}$. **[2]**

(ii) Hence specify the transformation equivalent to two applications of T. **[1]**

(c) In the case where $\lambda = 1$, T is equivalent to a transformation S followed by a reflection in the x-axis.

(i) Determine the matrix associated with S. **[3]**

(ii) Hence describe the transformation S. **[2]**

7 **(a)** **(i)** Find the modulus and argument of z_1, where $z_1 = 1+\mathrm{i}$. **[2]**

(ii) Given that $|z_2| = 2$ and $\arg(z_2) = \frac{1}{6}\pi$, express z_2 in $a+b\mathrm{i}$ form, where a and b are exact real numbers. **[2]**

(b) Using these results, find the exact value of $\sin\frac{5}{12}\pi$, giving the answer in the form $\frac{\sqrt{m}+\sqrt{n}}{p}$, where m, n and p are integers. **[5]**

8 **In this question you must show detailed reasoning.**

The equation $x^3 + kx^2 + 15x - 25 = 0$ has roots α, β and $\frac{\alpha}{\beta}$. Given that $\alpha > 0$, find, in any order,

- the roots of the equation,
- the value of k. **[7]**

9 **(a)** On a single Argand diagram, sketch the loci defined by

- $\arg(z-2) = \frac{3}{4}\pi$,
- $|z| = |z+2-\mathrm{i}|$. **[4]**

(b) **In this question you must show detailed reasoning.**

The point of intersection of the two loci in part **(a)** represents the complex number w.

Find w, giving your answer in exact form. **[5]**

1 The *specific energy* of a substance has SI unit $\mathrm{J\,kg^{-1}}$ (joule per kilogram).

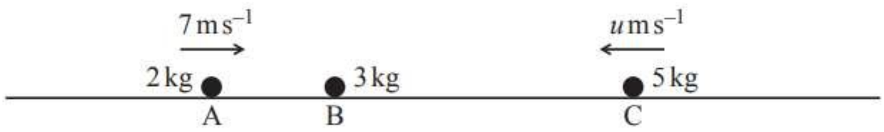
(a) Determine the dimensions of specific energy. **[2]**

A particular brand of protein powder contains approximately 345 Calories (Cal) per 4 ounce (oz) serving. An athlete is recommended to take 40 grams of the powder each day.

You are given that $1\,\mathrm{oz} = 28.35$ grams and $1\,\mathrm{Cal} = 4184\,\mathrm{J}$.

(b) Determine, in joules, the amount of energy in the athlete's recommended daily serving of the protein powder. **[2]**

3 Three small uniform spheres A, B and C have masses $2\,\mathrm{kg}$, $3\,\mathrm{kg}$ and $5\,\mathrm{kg}$ respectively. The spheres move in the same straight line on a smooth horizontal table, with B between A and C. Sphere A moves towards B with speed $7\,\mathrm{m\,s^{-1}}$, B is at rest and C moves towards B with speed $u\,\mathrm{m\,s^{-1}}$, as shown in the diagram.



Spheres A and B collide. Collisions between A and B can be modelled as perfectly elastic.

(a) Determine the magnitude of the impulse of A on B in this collision. **[5]**

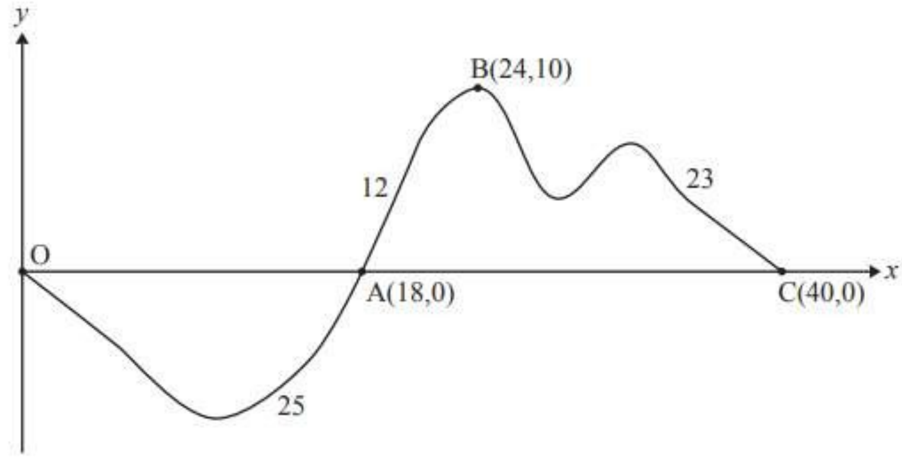
(b) Use this collision to verify that in a perfectly elastic collision no kinetic energy is lost. **[1]**

After the collision between A and B, sphere B subsequently collides with C. The coefficient of restitution between B and C is $\frac{1}{4}$.

(c) Show that, after the collision between B and C, B has a speed of $(1.225 - 0.78125u)\,\mathrm{m\,s^{-1}}$ towards C. **[4]**

(d) Determine the range of values for u for there to be a second collision between A and B. **[2]**

4 The diagram shows the path of a particle P of mass 2 kg as it moves from the origin O to C via A and B. The lengths of the sections OA, AB and BC are given in the diagram. The units of the axes are metres.



P, starting from O, moves along the path indicated in the diagram to C under the action of a constant force of magnitude T N acting in the positive x-direction. As P moves, it does R J of work for every metre travelled against resistances to motion.

It is given that

- the speed of P at O is $3\,\mathrm{m\,s^{-1}}$,
- the speed of P at A is $11\,\mathrm{m\,s^{-1}}$,
- the speed of P at C is $15\,\mathrm{m\,s^{-1}}$.

You should assume that both x- and y-axes lie in a horizontal plane.

(a) By considering the entire path of P from O to C, show that

$$20T - 30R = 108.$$ **[2]**

(b) By formulating a second equation, determine the values of T and R. **[3]**

(c) It is now given that the x-axis is horizontal, and the y-axis is directed vertically upwards. By considering the kinetic energy of P at B, show that the motion as described above is impossible. **[3]**

Mechanics b

1 The end O of a light elastic string OA is attached to a fixed point.

Fiona attaches a mass of 1 kg to the string at A. The system hangs vertically in equilibrium and the length of the stretched string is 70 cm.

Fiona removes the 1 kg mass and attaches a mass of 2 kg to the string at A. The system hangs vertically in equilibrium and the length of the stretched string is now 80 cm.

Fiona then removes the 2 kg mass and attaches a mass of 5 kg to the string at A. The system hangs vertically in equilibrium.

(a) Use the information given in the question to determine expected values for

- the length of the stretched string when the 5 kg mass is attached,
- the elastic potential energy stored in the string in this case. **[7]**

Fiona discovers that, when the mass of 5 kg is attached to the string at A, the length of the stretched string is greater than the expected length.

(b) Suggest a reason why this has happened. **[1]**

Review - Roots of Polynomials

Jan 06 FP1

5 (a) (i) Calculate $(2 + \mathrm{i}\sqrt{5})(\sqrt{5} - \mathrm{i})$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - \mathrm{i}$ is a root of the equation

$$(2 + \mathrm{i}\sqrt{5})\,z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - \mathrm{i}$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

Jan 07 FP1

1 (a) Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + \mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + \mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*

June 08 FP1

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

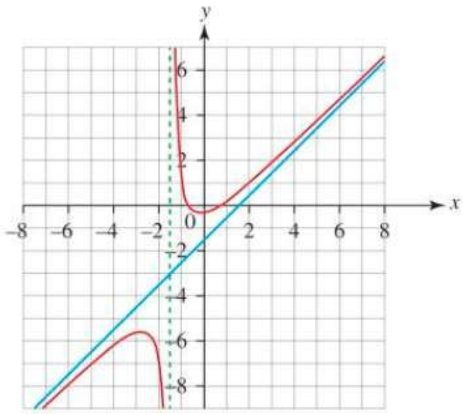
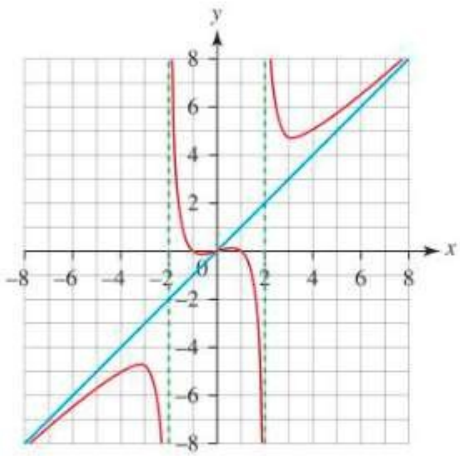
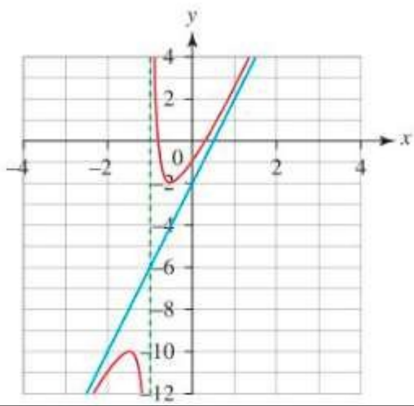
(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = -\dfrac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$. *(2 marks)*

CONSOLIDATION ANSWERS

	Oblique asymptotes	Complex powers and roots
1	$y=-2x-2$ and $x=1$	$4\sqrt{5}e^{-i2.03}$ $\quad 4e^{-\frac{2\pi}{3}i}$ $\quad 3e^{-\frac{5\pi}{6}i}$ $\frac{7}{2}-\frac{7\sqrt{3}}{2}i$ **a** 1 **b** 25 **c** $\frac{\pi}{7}$ **d** $\frac{3\pi}{7}$
2	**b** Asymptotes at $x=-\frac{3}{2}$ and $y=x-\frac{3}{2}$ 	**a** -16 **b** $64i$ **c** $\frac{1}{4}i$ **d** $\frac{1}{16}$ **a** $8-8\sqrt{3}i$ **b** $-\frac{1}{8}i$ **c** $\frac{1}{8}+\frac{\sqrt{3}}{8}i$ **d** $-\frac{1}{8}$
3	Asymptotes at $x=\pm2$ and $y=x$ 	**a** $2e^{\frac{\pi}{12}i}, 2e^{\frac{3\pi}{4}i}, 2e^{-\frac{7\pi}{12}i}$ **b** $2e^{\frac{\pi}{4}i}, 2e^{\frac{11\pi}{12}i}, 2e^{-\frac{5\pi}{12}i}$ **c** $2e^{-\frac{\pi}{4}i}, 2e^{\frac{5\pi}{12}i}, 2e^{-\frac{11\pi}{12}i}$ **d** $2e^{-\frac{\pi}{12}i}, 2e^{\frac{7\pi}{12}i}, 2e^{-\frac{3\pi}{4}i}$
R	**a** $a=4, b=2$ **b** $x=-1$ **c** $\left(-\frac{1}{2},-2\right),\left(-\frac{3}{2},-10\right)$ **d** 	$z=e^{\theta i}$ **a** $z^n+\frac{1}{z^n}=(e^{\theta i})^n+\frac{1}{(e^{\theta i})^n}$ $=e^{n\theta i}+e^{-n\theta i}$ $=2\cos(n\theta)$ as required **b** $z^n-\frac{1}{z^n}=(e^{\theta i})^n-\frac{1}{(e^{\theta i})^n}$ $=e^{n\theta i}-e^{-n\theta i}$ $=2i\sin(n\theta)$ as required **a** $1+\omega+\omega^2=\frac{(1-\omega^3)}{1-\omega}$ $=\frac{(1-1)}{1-\omega}=0$ **b** **i** 0 **ii** 1 **iii** -1 **iv** 0
C		**a** $\frac{3\sqrt{3}}{2}-\frac{3}{2}i, 3i, -\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ **b** **i** $\frac{27\sqrt{3}}{4}$ square units **ii** $3\sqrt{3}(\sqrt{2}+1)$ units

<table>
<tr><th></th><th>Further vectors</th><th>Further matrices</th></tr>
<tr><td>1</td><td>

a $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{c}=-\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{c}$

$=\mathbf{b}\times(\mathbf{c}-\mathbf{a})+\mathbf{a}\times\mathbf{c}$

$=(\mathbf{b}+\mathbf{a})\times(\mathbf{c}-\mathbf{a})$ (since $\mathbf{a}\times\mathbf{a}=0$)

b $\mathbf{b}\times(2\mathbf{a}+\mathbf{c})-\mathbf{c}\times(\mathbf{b}-\mathbf{c})=\mathbf{b}\times(2\mathbf{a})+\mathbf{b}\times\mathbf{c}-\mathbf{c}\times\mathbf{b}-\mathbf{c}\times(-\mathbf{c})$

$=2\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{c}$

$=2\mathbf{b}\times\mathbf{a}+2\mathbf{b}\times\mathbf{c}$

$=2\mathbf{b}\times(\mathbf{a}+\mathbf{c})$

$$\begin{pmatrix} 0 \\ -\frac{3}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \end{pmatrix}$$

$b=-3$

$a=1$

$c=6$

$$\frac{\sqrt{2}}{2}$$

</td><td>

$12b-15ab$

$$\begin{pmatrix} \frac{2}{a} & -\frac{3}{a} & \frac{4}{a} \\ 5 & -7 & 9 \\ -1 & 2 & -2 \end{pmatrix}$$

a For example, $\begin{vmatrix} 2 & 1 & -3 \\ 4 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}=\begin{vmatrix} 0 & 1 & -3 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix}$ C1−2C2

b −7

a $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=\begin{vmatrix} a+b & b+c & c+a \\ b & c & a \\ c & a & b \end{vmatrix}$ R1+R2

$=\begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$ R1+R3

$=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$=(a+b+c)[(bc-a^2)-(b^2-ac)+(ab-c^2)]$

$=(a+b+c)(ab+ac+bc-a^2-b^2-c^2)$

as required

$\begin{vmatrix} a^2 & b^2 & c^2 \\ bc & ca & ab \\ 1 & 1 & 1 \end{vmatrix}=\begin{vmatrix} a^2 & b^2-a^2 & c^2 \\ bc & ca-bc & ab \\ 1 & 0 & 1 \end{vmatrix}$ C2−C1

$=\begin{vmatrix} a^2 & b^2-a^2 & c^2-a^2 \\ bc & ca-bc & ab-bc \\ 1 & 0 & 0 \end{vmatrix}$ C3−C1

$=\begin{vmatrix} a^2 & (b-a)(b+a) & c^2-a^2 \\ bc & -c(b-a) & ab-bc \\ 1 & 0 & 0 \end{vmatrix}$

$=(b-a)\begin{vmatrix} a^2 & b+a & c^2-a^2 \\ bc & -c & ab-bc \\ 1 & 0 & 0 \end{vmatrix}$

$=(b-a)\begin{vmatrix} a^2 & b+a & (c-a)(c+a) \\ bc & -c & -b(c-a) \\ 1 & 0 & 0 \end{vmatrix}$

$=(b-a)(c-a)\begin{vmatrix} a^2 & b+a & c+a \\ bc & -c & -b \\ 1 & 0 & 0 \end{vmatrix}$

$=(b-a)(c-a)[-b(b+a)--c(c+a)]$

$=(b-a)(c-a)(-b^2-ab+c^2+ac)$

$=(b-a)(c-a)(c-b)(a+b+c)$ as required

$\begin{vmatrix} a+b & a+c & b+c \\ c & b & a \\ c^2 & b^2 & a^2 \end{vmatrix}=\begin{vmatrix} a+b+c & a+c+b & b+c+a \\ c & b & a \\ c^2 & b^2 & a^2 \end{vmatrix}$ R1+R2

$=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c & b & a \\ c^2 & b^2 & a^2 \end{vmatrix}$

$=(a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ c-b & b & a \\ c^2-b^2 & b^2 & a^2 \end{vmatrix}$ C1−C2

$=(a+b+c)(c-b)\begin{vmatrix} 0 & 1 & 1 \\ 1 & b & a \\ c+b & b^2 & a^2 \end{vmatrix}$

</td></tr>
</table>

		$=(a+b+c)(c-b)\begin{vmatrix}0&0&1\\1&b-a&a\\c+b&b^2-a^2&a^2\end{vmatrix}$ C2−C3 $=(a+b+c)(c-b)(b-a)[(a+b)-(c+b)]$ $=(a+b+c)(c-b)(b-a)(a-c)$ as required **d** $\begin{vmatrix}a&-b&c\\c-b&a+c&a-b\\-bc&ac&-ab\end{vmatrix}=\begin{vmatrix}a+b&-b&c\\-b-a&a+c&a-b\\-bc-ac&ac&-ab\end{vmatrix}$ C1−C2 $=\begin{vmatrix}a+b&-b&c\\-(a+b)&a+c&a-b\\-c(a+b)&ac&-ab\end{vmatrix}$ $=(a+b)\begin{vmatrix}1&-b&b+c\\-1&a+c&-b-c\\-c&ac&-ab-ac\end{vmatrix}$ C3−C2 $=(a+b)\begin{vmatrix}1&-b&b+c\\-1&a+c&-(b+c)\\-c&ac&-a(b+c)\end{vmatrix}$ $=(a+b)(b+c)\begin{vmatrix}1&-b&1\\-1&a+c&-1\\-c&ac&-a\end{vmatrix}$ $=(a+b)(b+c)\begin{vmatrix}0&-b&1\\0&a+c&-1\\-c+a&ac&-a\end{vmatrix}$ C1−C3 $=(a+b)(b+c)(c-a)\begin{vmatrix}0&-b&1\\0&a+c&-1\\-1&ac&-a\end{vmatrix}$ $=(a+b)(b+c)(c-a)[-1(b-(a+c))]$ $=(a+b)(b+c)(c-a)(a-b+c)$ as required
2	**a** $\mathbf{r}=\begin{pmatrix}-2\\0\\0\end{pmatrix}+s\begin{pmatrix}7\\1\\1\end{pmatrix}+t\begin{pmatrix}8\\2\\-5\end{pmatrix}$ **b** $\mathbf{n}=\begin{pmatrix}7\\1\\1\end{pmatrix}\times\begin{pmatrix}8\\2\\-5\end{pmatrix}=\begin{pmatrix}-7\\43\\6\end{pmatrix}$ $\begin{pmatrix}-2\\0\\0\end{pmatrix}\cdot\begin{pmatrix}-7\\43\\6\end{pmatrix}=14$ $\mathbf{r}\cdot(-7\mathbf{i}+43\mathbf{j}+6\mathbf{k})=14$ **c** $-7x+43y+6z-14=0$ $\quad$ 21.1°	If we multiply the first equation by 2 we get $6x-4y+2z=14$ so this plane is parallel to the second as they are the same except the constant term. Therefore, the three planes do not meet. Intersect at $(2,-5,-3)$ $\det\begin{pmatrix}1&0&2\\-2&1&4\\9&-2&2\end{pmatrix}=1(12--8)+2(4-9)$ $=0$ so not a unique solution First equation gives $x=1-2z$ Substitute into second equation to give $-2(1-2z)+y+4z=0\Rightarrow y=2-8z$ Check in third equation: $9(1-2z)+2(2-8z)+2z=5$ So they form a sheaf (the line has equations such as $x=1-2z,\ y=2-8z$)
3	$\frac{7}{6}$ units $(11, 12, -5)$	$\lambda=-3,\ 1,\ 5$, corresponding eigenvectors are $\begin{pmatrix}5\\-3\\4\end{pmatrix},\begin{pmatrix}1\\-1\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\mathbf{B}=\begin{pmatrix}-1&6\\1&1\end{pmatrix}\begin{pmatrix}-4&0\\0&3\end{pmatrix}\begin{pmatrix}-\frac{1}{7}&\frac{6}{7}\\\frac{1}{7}&\frac{1}{7}\end{pmatrix}$

R	**a** **i** $\left\lvert\dfrac{-20\sqrt{51}}{51}\right\rvert$ **ii** $\left\lvert\dfrac{-20\sqrt{51}}{51}\right\rvert$ **b** Since A and B are the same distance from Π and are the same side of Π the line l must be parallel to the plane therefore it doesn't intersect it. $\begin{pmatrix}-3\\6\\-9\end{pmatrix}=\frac{3}{2}\begin{pmatrix}-2\\4\\-6\end{pmatrix}$ so they are parallel (4, −3, 8) satisfies B so check A: $\frac{4-3}{-2}=-\frac{1}{2}$ $\frac{-3+1}{4}=-\frac{1}{2}$ $\frac{8-5}{-6}=-\frac{1}{2}$ so (4, −3, 8) satisfies both equations therefore they represent the same line.	**a** −128 since rows 1 and 2 swapped **c** 128 since row 2 added to row 1 **e** $128\times3=384$ since column 2 multiplied by 3 **g** 256 since columns 1 and 3 then columns 2 and 3 swapped and column 2 doubled
C	$=\dfrac{27}{2}\sqrt{6}$ square units	**a** $\begin{pmatrix}5^4 & -2(5^4)\\-2(5^4) & 4(5^4)\end{pmatrix}$ **b** $5^{n-1}\begin{pmatrix}1 & -2\\-2 & 4\end{pmatrix}$

Roots of polynomials, answers:

Jan 06 FP1

5(a)(i)	Full expansion of product Use of $i^2=-1$ $(2+\sqrt{5}i)(\sqrt{5}-i)=3\sqrt{5}+3i$	M1 m1 A1	3	$\sqrt{5}\sqrt{5}=5$ must be used – Accept not fully simplified
(ii)	$z^*=x-iy\ (=\sqrt{5}+i)$ Hence result	M1 A1	2	Convincingly shown (AG)
(b)(i)	Other root is $\sqrt{5}+i$	B1	1	
(ii)	Sum of roots is $2\sqrt{5}$ Product is 6	B1 M1A1	3	
(iii)	$p=-2\sqrt{5}$, $q=6$	B1 B1√	2	ft wrong answers in (ii)
	Total		**11**	

Jan 07 FP1

1(a)(i)	Roots are $\pm 4i$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1\pm 4i$	M1A1	2	M1 for correct method
(b)(i)	$(1+x)^3=1+3x+3x^2+x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1+i)^3=1+3i-3-i=-2+2i$	M1A1	2	M1 if $i^2=-1$ used
(iii)	$(1+i)^3+2(1+i)-4i$ $\ldots=(-2+2i)+(2-2i)=0$	M1 A1	2	with attempt to evaluate convincingly shown (**AG**)
	Total		**10**	

June 08 FP1

1(a)	$\alpha + \beta = -1,\ \alpha\beta = 5$	B1B1	2	
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		with numbers substituted
	$\ldots = 1 - 10 = -9$	A1F	2	ft sign error(s) in (a)
(c)	$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$	M1		
	$\ldots = -\dfrac{9}{5}$	A1	2	AG: A0 if $\alpha + \beta = 1$ used
(d)	Product of new roots is 1	B1		PI by constant term 1 or 5
	Eqn is $5x^2 + 9x + 5 = 0$	B1F	2	ft wrong value for product
	Total		**8**	